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ON IDENTIFICATION PROBLEM OF LINEAR INTERVAL DYNAMICAL SYSTEM

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Abstract. We consider identification problem for linear dynamical systems with interval coefficients of corresponding matrices. Using concepts of universal and sub-universal solutions of linear systems we obtain methods for construction identifiable parameter and get its properties.

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1. Introduction

The classical theory of optimal control and corresponding particular problems were introduced and developed in 1960-s by L.S.Pontryagin, R. Kalman, V.G.Boltyanski, R.V.Gamkrelidze, N.N.Krasovsky and other researchers [1, 5, 6, 7, 8, 9]. Identification problem is regarded as one of the most important for control theory and in general it means the determining of parameters of the mathematical models on the basis of experimental data, observations, or other additional information about the object of control.

The statement of the problem is: given a mathematical model

$$\dot{x} = A(t)x + B(t)w, \; x(t_0) = 0 \tag{1}$$

of movement of an object and the results $y(t)$ of the observation of its trajectory $x(t)$:

$$y(t) = C(t)x(t), \; t_0 \leq t \leq t_1. \tag{2}$$

Here $A(t)$, $B(t)$, $C(t)$ are continuous for R matrix functions of sizes $n \times n$, $n \times r$, $m \times r$. Accordingly, $m < n$; $y(t)$ is continuous on the $[t_0, t_1]$ vector function, and w is an unknown vector of parameters.

It is required

1) to determine under what conditions on $A(t), B(t), C(t), y(t), x_0, t_0, t_1$ there exists a vector of parameters $w \in R^r$ for which the corresponding solution $x(t)$ of the Cauchy problem (1) satisfies the equality (2);

2) in the case of the existence of a vector w, to specify the procedure for it to be determined.

In actual applications, vector w and function $y(t)$ are treated as the input and output of the model (1). In these terms, the problem of identification consists of finding the input of the model by providing a desired output.

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We refer to a triple $x(t), y(t), w$, satisfying the conditions (1), (2) of identification problem as a *process*. Each process is unambiguously determined using a vector of parameters w.

Following to [1, 8, 9], we say that the system (1), (2) is *identifiable in a direction q (q-identifiable)* if there exists a continuous vector-function $z(t) : [t_0, t_1] \to R^m$ such that for any process $x(t), y(t), w$, the below equality holds

$$w'q = \int_{t_0}^{t_1} y(t)'z(t)dt. \tag{3}$$

In other words, in a q-identifiable system, we can restore the projection of each vector w on the direction q by using one and the same linear operation that corresponds to the w process.

Solution of identification problem has been obtained by L.S.Pontryagin and etc. [9]. The following theorem gives the criterion of identification.

Theorem 1.1. (Criterion of Identification). *The system (1), (2) is identifiable in direction q if and only if the system of linear algebraic equations*

$$W_3(t_0, t_1)p = q \tag{4}$$

with a matrix of coefficients

(5)

$$W_3(t_0, t_1) = \int_{t_0}^{t_1} \left(\int_{t_0}^{t} B(\tau)'F(t, \tau)'d\tau \right) C(t)'C(t) \left(\int_{t_0}^{t} F(t, \tau)B(\tau)d\tau \right) dt$$

has a solution.

The computation for matrix $W_3(t_0, t_1)$ can be reduced to the solution of the matrix Cauchy problem

$$\dot{V} = A(t)V + B(t),\ \dot{W} = V'C(t)'C(t)V,$$
$$V(t_0) = 0,\ W(t_0) = 0$$

for functions

$$V(t) = \int_{t_0}^{t} F(t, \tau)B(\tau)d\tau,\ W(t) = \int_{t_0}^{t} V(\tau)'C(\tau)'C(\tau)V(\tau)d\tau = W_3(t_0, t).$$

Restoring the parameter vector includes the following procedure: suppose the system (1), (2) is identifiable in any direction q. Then, the matrix of coefficients (5) of equations (4) has a maximum rank r and as a consequence, there is an inverse matrix $W_3^{-1}(t_0, t_1)$. By definition

$$E = W_3^{-1}(t_0, t_1)W_3(t_0, t_1).$$

We multiply this identity by sought-for vector w. Using the notation (3), (5), we obtain

$$w = W_3^{-1}(t_0, t_1) \int_{t_0}^{t_1} \left(\int_{t_0}^{t} B(\tau)'F(t, \tau)'d\tau \right) C(t)'y(t)dt.$$

This formula expresses a vector of parameters w via the known data of the problem of identification.

This paper is devoted to the solving of identification problem for linear system (1) - (2) with interval coefficients of matrices $A(t)$, $B(t)$ and $C(t)$.

First problem that we have to solve is the problem of getting solutions of an interval linear system.

2. Auxiliary problem

We consider an interval linear system

$$\sum_{j=1}^{n} d_{ij} z_j = f_i, i = 1, 2, ..., m,$$

where $\underline{d}_{ij} \le d_{ij} \le \overline{d}_{ij}$, $\underline{f}_i \le f_i \le \overline{f}_i$. Or in matrix form

$$Dz = f, \quad (6)$$

$$\underline{D} \le D \le \overline{D}, \underline{f} \le f \le \overline{f}.$$

Problem: We need to determine vector $z \in R^n$ satisfying system (6) for any matrices D and f, where $\underline{D} \le D \le \overline{D}$, $\underline{f} \le f \le \overline{f}$.

According to [2, 4], we introduce non-negative vector ε as a discrepancy between vector Dz and vector f:

$$|Dz - f| \le \varepsilon.$$

Then

Definition. *Vector $z \in R^n$ is called an ε-solution of the system (6) if*

$$f - \varepsilon \le Dz \le f + \varepsilon$$

for any matrices D: $\underline{D} \le D \le \overline{D}$ and f: $\underline{f} \le f \le \overline{f}$.

Note that ε-solution always exists even when system (6) is not consistent.

We introduce the following representation of intervals for components of matrix D: if we let

$$d_{ij} = \underline{d}_{ij} + \lambda(\overline{d}_{ij} - \underline{d}_{ij}), 0 \le \lambda \le 1, d_{ij}^{\Delta} = (\overline{d}_{ij} - \underline{d}_{ij}) \quad (7)$$

then

(8)

$$\max_{\underline{d}_{ij} \le d_{ij} \le \overline{d}_{ij}} d_{ij} z_j = \max_{0 \le \lambda \le 1} (\underline{d}_{ij} + \lambda(\overline{d}_{ij} - \underline{d}_{ij})) z_j = \max_{0 \le \lambda \le 1} \left(\underline{d}_{ij} z_j + \lambda d_{ij}^{\Delta} z_j\right)$$

$$= \max_{0 \le \lambda \le 1} \underline{d}_{ij} z_j + d_{ij}^{\Delta} \max_{0 \le \lambda \le 1} \lambda z_j = \underline{d}_{ij} z_j + \tfrac{1}{2} d_{ij}^{\Delta} (z_j + |z_j|) = (d_{ij} + \tfrac{1}{2} d_{ij}^{\Delta}) z_j + \tfrac{1}{2} d_{ij}^{\Delta} |z_j|,$$

(9)

$$\min_{\underline{d}_{ij} \le d_{ij} \le \overline{d}_{ij}} d_{ij} z_j = \min_{0 \le \lambda \le 1} (\underline{d}_{ij} + \lambda(\overline{d}_{ij} - \underline{d}_{ij})) z_j = \min_{0 \le \lambda \le 1} \left(\underline{d}_{ij} z_j + \lambda d_{ij}^{\Delta} z_j\right)$$

$$= \min_{0 \le \lambda \le 1} \underline{d}_{ij} z_j + d_{ij}^{\Delta} \min_{0 \le \lambda \le 1} \lambda z_j = \underline{d}_{ij} z_j + \tfrac{1}{2} d_{ij}^{\Delta} (z_j - |z_j|) = (d_{ij} + \tfrac{1}{2} d_{ij}^{\Delta}) z_j - \tfrac{1}{2} d_{ij}^{\Delta} |z_j|.$$

And the Theorem 2.1 holds.

Theorem 2.1. *Vector $z \in R^n$ is ε-solution of system (6) if and only if*

$$\sum_{j=1}^{n} (\underline{d}_{ij} + \tfrac{1}{2} d_{ij}^{\Delta}) z_j + \tfrac{1}{2} \sum_{j=1}^{n} d_{ij}^{\Delta} |z_j| \le \underline{f}_i + \varepsilon_i,$$

$$\sum_{j=1}^{n} (\underline{d}_{ij} + \tfrac{1}{2} d_{ij}^{\Delta}) z_j - \tfrac{1}{2} \sum_{j=1}^{n} d_{ij}^{\Delta} |z_j| \ge \overline{f}_i - \varepsilon_i, i = 1, 2, ..., m.$$

In more convenient matrix form inequalities of this theorem becomes

$$\begin{aligned}\left(\underline{D}+\tfrac{1}{2}D^{\Delta}\right)z+\tfrac{1}{2}D^{\Delta}\,|z| &\le \underline{f}+\varepsilon,\\ \left(\underline{D}+\tfrac{1}{2}D^{\Delta}\right)z-\tfrac{1}{2}D^{\Delta}\,|z| &\ge \overline{f}-\varepsilon.\end{aligned} \tag{10}$$

Here $\underline{D}$ and $D^{\Delta}=\left(\overline{D}-\underline{D}\right)$ are $m\times n$ matrices of corresponding entries $\underline{d}_{ij}$ and $d_{ij}^{\Delta}=\overline{d}_{ij}-\underline{d}_{ij}$. Here and further we understand all matrix and vector operations component-wise.

Due to the notation (7), theorem 2.1 and inequalities (10), we get the following extreme problem

$$\begin{gathered} e^{T}\varepsilon\to\min\\ \left(\underline{D}+\tfrac{1}{2}D^{\Delta}\right)z+\tfrac{1}{2}D^{\Delta}\,|z|\le\underline{f}+\varepsilon\\ \left(\underline{D}+\tfrac{1}{2}D^{\Delta}\right)z-\tfrac{1}{2}D^{\Delta}\,|z|\ge\overline{f}-\varepsilon\\ \varepsilon\ge 0.\end{gathered} \tag{11}$$

Here e^{T} means the row (1, 1, ... , 1) of n components.

Introduction of variable $s=|z|$ allows us to reduce non-linear problem (11) to the linear problem (12)

$$\begin{gathered} e^{T}\varepsilon\to\min\\ \left(\underline{D}+\tfrac{1}{2}D^{\Delta}\right)z+\tfrac{1}{2}D^{\Delta}s\le\underline{f}+\varepsilon\\ \left(\underline{D}+\tfrac{1}{2}D^{\Delta}\right)z-\tfrac{1}{2}D^{\Delta}s\ge\overline{f}-\varepsilon\\ -s\le z\le s\\ \varepsilon\ge 0.\end{gathered} \tag{12}$$

Solution of the linear problem (12) is a triple $(z^{*},\varepsilon^{*},s^{*})$. z^{*} is a universal solution and ε^{*} is minimum discrepancy.

Note that a universal solution always exists as well as an ε-solution. Problem (12) has at least one solution: $z=0$ and $\varepsilon\ge\max\left\{\left|\overline{f}\right|,\left|\underline{f}\right|\right\}$.

3. Identification problem for interval system

Consider interval linear system

$$\dot{x}=A(t)x+B(t)w,x(t_0)=x^{0} \tag{13}$$

with homogeneous equation of interval observation

$$y(t)=C(t)x(t),t_0\le t\le t_1, \tag{14}$$

where

$$\underline{A}(t)\le A(t)\le\overline{A}(t),\underline{B}(t)\le B(t)\le\overline{B}(t),\underline{C}(t)\le C(t)\le\overline{C}(t). \tag{15}$$

Here $x(t)\in R^{n}$ is a state variable, x^{0} - initial state, w - sought-for r-vector of parameters, $y(t)$- m-vector of the results of the measurement of a phase variable $x(t)$ and t_0, t_1 are initial and terminal moments of time. The length of intervals $\underline{A}(t)\le A(t)\le\overline{A}(t)$, $\underline{B}(t)\le B(t)\le\overline{B}(t)$, $\underline{C}(t)\le C(t)\le\overline{C}(t)$ we treat as level of uncertainty.

Let $A_0(t)=\frac{1}{2}(\underline{A}(t)+\overline{A}(t))$, $A_{\Delta}(t)=\frac{1}{2}(\overline{A}(t)-\underline{A}(t))$ be center and radius of interval $\underline{A}(t)\le A(t)\le\overline{A}(t)$ accordingly. Analogously, we have for matrices $B(t)$ and $C(t)$.

Interval statement of the identification problem doesn't mean the exact restoring of vector w. So our purpose is to obtain the "best" in some sense estimation of vector w that satisfies to given observation $y(t)$ on interval $t_0 \le t \le t_1$ using known center and radius of interval matrices $A(t)$, $B(t)$, $C(t)$. The accuracy of such estimation, evidently, is related with the level of uncertainty of the system (13) described, for instance, by the maximum of the norms $\|A_\Delta(t)\|$, $\|B_\Delta(t)\|$, $\|C_\Delta(t)\|$ on $t_0 \le t \le t_1$.

We fix admissible matrices $A(t)$, $B(t)$, $C(t)$ from corresponding intervals (15) and on interval $t_0 \le t \le t_1$ describe the dynamic of the system (13) by Cauchy formula

$$x(t) = F(t,t_0)x^0 + \int_{t_0}^{t} F(t,\tau)B(\tau)w d\tau = F(t,t_0)x^0 + K(t)w. \tag{16}$$

Here $F(t,\tau)$ is a fundamental matrix of solutions for homogeneous system $\dot{x} = A(t)x$ and

$$K(t) = \int_{t_0}^{t} F(t,\tau)B(\tau)d\tau.$$

Substituting (16) into (14) yields the relation of known observations $y(t)$ with unknown coefficients $A(t)$, $B(t)$, $C(t)$ and parameter w:

$$y(t) = C(t)x(t) = C(t)\left(F(t,t_0)x^0 + K(t)w\right), t_0 \le t \le t_1. \tag{17}$$

Let further $F_0(t,\tau)$ be a fundamental matrix of solutions for the central homogeneous system $\dot{x} = A_0(t)x$ and $K_0(t) = \int_0^t F_0(t,\tau)B_0(\tau)d\tau$. We apply linear integral mapping with the kernel $K_0(t)'C_0(t)'$ and get a system of linear algebraic equations

$$Vw = g \tag{18}$$

with respect to vector w and matrix coefficients

$$V = \int_{t_0}^{t_1} K_0(t)'C_0(t)'C(t)K(t)dt, \tag{19}$$

$$g = \int_{t_0}^{t_1} K_0(t)'C_0(t)'\left(y(t) - C(t)F(t,t_0)x^0\right)dt.$$

As we can see from the formula (19), matrix V and vector g of the sizes $n \times n$ and $n \times 1$ depends on uncertain matrices $A(t)$, $B(t)$, $C(t)$ and are, essentially, unknown.

Using obtained in [3] inequality

$$|F(t,\tau) - F_0(t,\tau)| \le F(t,\tau) - F_{|0|}(t,\tau) \equiv F_\Delta(t,\tau), \tag{20}$$

we construct external estimation:

$$\begin{aligned} &|K(t) - K_0(t)| = \left|\int_{t_0}^t F(t,\tau)B(\tau)d\tau - \int_{t_0}^t F_0(t,\tau)B_0(\tau)d\tau\right| \le \\ &\int_{t_0}^t |F(t,\tau)B(\tau) - F_0(t,\tau)B_0(\tau)|\, d\tau \le \\ &\int_{t_0}^t |(F(t,\tau) - F_0(t,\tau))\,B(\tau) + F_0(t,\tau)(B(\tau) - B_0(\tau)|\, d\tau \le \\ &\int_{t_0}^t \left(F_\Delta(t,\tau)\left(|B_0(\tau)| + B_\Delta(\tau)\right) + |F_0(t,\tau)|\, B_\Delta(\tau)\right) d\tau = K_\Delta(t). \end{aligned} \tag{21}$$

Returning to (18) and (19) and taking

$$V_0 = \int_{t_0}^{t_1} K_0(t)'C_0(t)'C_0(t)K_0(t)dt,$$

$$g_0 = \int_{t_0}^{t_1} K_0(t)' C_0(t)' \left(y(t) - C_0(t) F_0(t,t_0) x^0 \right) dt,$$

we get external interval evaluations

$$|V - V_0| \leq V_\Delta, |g - g_0| \leq g_\Delta \tag{22}$$

with known matrix

(23)
$$V_\Delta = \int_{t_0}^{t_1} \left| K_0(t)' C_0(t)' \right| \left[|C_0(t)| K_\Delta(t) + C_\Delta(t) |K_0(t)| + C_\Delta(t) K_\Delta(t) \right] dt,$$

and vector

(24)
$$g_\Delta = \int_{t_0}^{t_1} \left| K_0(t)' C_0(t)' \right| \left(\left(|C_0(t)| + C_\Delta(t) \right) F_\Delta(t,t_0) + C_\Delta(t) \left| F_0(t,t_0) \right| \right) \left| x^0 \right| dt.$$

Thus, by mean of external estimation non-linear system (18), (19) was transformed to linear system (18), (22).

Let

$$\underline{V} = V_0 - V_\Delta, \overline{V} = V_0 + V_\Delta, \underline{g} = g_0 - g_\Delta, \overline{g} = g_0 + g_\Delta,$$

then we can rewrite inequalities (22) in the form $\underline{V} \leq V \leq \overline{V}$ and $\underline{g} \leq g \leq \overline{g}$.

Applying the concept of universal solution and problem (12), we form an extreme problem

$$\begin{array}{l} e^T \varepsilon \to \min \\ (\underline{V} + \frac{1}{2} V^\Delta) w + \frac{1}{2} V^\Delta s - \varepsilon \leq \underline{g} \\ (\underline{V} + \frac{1}{2} V^\Delta) w - \frac{1}{2} V^\Delta s + \varepsilon \leq \overline{g} \\ -s \leq w \leq s \\ \varepsilon \geq 0. \end{array} \tag{25}$$

Optimal solution of the linear programming problem (25) gives a solution of identification problem (13) - (15).

Note that due to consistency of the problem (25), the construction of universal solutions formally doesn't demand on any assumptions about properties of observation. Nevertheless, from the practical point of view the wider the intervals of uncertainty (15) the rougher will be the estimation of identifying parameter w. In particular, for degenerate matrices ($A_0(t) \equiv A_\Delta(t) \equiv 0$, or $B_0(t) \equiv B_\Delta(t) \equiv 0$, or $C_0(t) \equiv C_\Delta(t) \equiv 0$) the restrictions of the problem (25) don't depend on vectors w and s and are reduced to the inequality $\varepsilon \geq 0$ and the optimal discrepancy is $\varepsilon^* = 0$.

Therefore, in many cases instead of universal solution it is more useful to utilize more rough but easer sub-universal solution defined from central system

$$\dot{x} = A_0(t) x(t) + B_0(t) w, y(t) = C_0(t) x(t), t_0 \leq t \leq t_1$$

in assumption of non-degeneracy of the matrix V_0 ($rank V_0 = n$) [2].

Notice that sub-universal solution is an approximation of a universal solution.

Using sub-universal solution

(26) $$\hat{w} = V_0^{-1} g_0$$

we determine the estimation of the discrepancy

(27) $$\hat{\varepsilon} = |V\hat{w} - g| \leq |V - V_0|\,|\hat{w}| + |g - g_0| \leq V_\Delta\,|\hat{w}| + g_\Delta = \varepsilon(\hat{w})$$

for any admissible matrices $A(t)$, $B(t)$, $C(t)$.

The discrepancy (27) decreases with continuous by $t_0 \leq t \leq t_1$ decreasing of the norm of matrices $A_\Delta(t)$, $B_\Delta(t)$, $C_\Delta(t)$ and in the limit (when $A_\Delta(t) \equiv 0$, $B_\Delta(t) \equiv 0$, $C_\Delta(t) \equiv 0$) it becomes zero. In this case the formula (26) gives the exact value of vector w.

We note that if $x(t_0) = x^0 = 0$ then the system (18) arrive at

$$Vw = \bar{g}_0$$

with matrix V of the form (19) and vector

$$\bar{g}_0 = \int_{t_0}^{t_1} K_0(t)' C_0(t)' y(t) dt$$

that is unambiguously defined by $y(t)$. In this case the evaluation (27) of the accuracy $\hat{\varepsilon}$ on sub-universal solution $\hat{w} = V_0^{-1}\bar{g}_0$ corresponds to inequality

$$\hat{\varepsilon} \leq V_\Delta\,|\hat{w}|\,.$$

Besides, it is true the following estimation

$$|w - \hat{w}| = \left|w - V_0^{-1} V w\right| \leq \left|V_0^{-1}\left(V_0 - V\right) w\right| \leq \left|V_0^{-1}\right| V_\Delta\,|w|$$

for any fixed matrix V from (22).

4. Simultaneous identification with initial vector

Let in the system (13) - (15) initial state x^0 be unknown and it demands identification along with vector w. We form modular matrix

$$M(t) = (F(t, t_0), K(t))$$

of sizes $n \times (n \times r)$ and modular column-vector on unknowns $z = (x^0, w)^T$of size $(n \times r) \times 1$. From (16), (17) we get

(28) $$y(t) = C(t)x(t) = C(t)M(t)z, t_0 \leq t \leq t_1.$$

Applying to (28) linear integral mapping with kernel $M_0(t)'C_0(t)'$ and matrix $M_0(t) = (F_0(t, t_0), K_0(t))$, by analogy with item 3, we construct the system of linear algebraic equations

(29) $$\tilde{V} w = \tilde{g}_0$$

with respect to variable z with matrix coefficients

(30) $$\tilde{V} = \int_{t_0}^{t_1} M_0(t)' C_0(t)' C(t) M(t) dt, \tilde{g}_0 = \int_{t_0}^{t_1} M_0(t)' C_0(t)' y(t) dt.$$

From (13) and (21) it follows that matrix $M(t) = (F(t, t_0), K(t))$ satisfies the estimation

$$|M(t) - M_0(t)| \leq M_\Delta(t) = (F_\Delta(t, t_0), K_\Delta(t))\,.$$

By analogy with (22), (23), we get

$$\left|\tilde{V}-\tilde{V}_0\right| \le \tilde{V}_\Delta = \int_{t_0}^{t_1} |M_0(t)'C_0(t)'| \left[|C_0(t)|\, M_\Delta(t) + C_\Delta(t)\, |M_0(t)| + C_\Delta(t)M_\Delta(t)\right] dt. \tag{31}$$

Following to (22) - (25), we determine universal solution of the system (29) - (31) solving the linear programming problem

$$\begin{aligned}
&e^T\varepsilon \to \min\\
&(\underline{\tilde{V}} + \tfrac{1}{2}\tilde{V}^\Delta)z + \tfrac{1}{2}\tilde{V}^\Delta s - \varepsilon \le \tilde{g}_0\\
&(\underline{\tilde{V}} + \tfrac{1}{2}\tilde{V}^\Delta)z - \tfrac{1}{2}\tilde{V}^\Delta s + \varepsilon \le \tilde{g}_0\\
&-s \le z \le s\\
&\varepsilon \ge 0
\end{aligned}$$

with vector of auxiliary variables s, as well as, sub-universal solution

$$\tilde{z} = \tilde{V}_0^{-1}\tilde{g}_0$$

assuming non-degeneracy of the matrix $\tilde{V}_0$.

By virtue of unambiguous definition of the vector $\tilde{g}_0 = \int_{t_0}^{t_1} M_0(t)'C_0(t)'y(t)dt$ the following estimation is true

$$\hat{\varepsilon} \le \tilde{V}_\Delta\, |\tilde{z}|\,, |z-\tilde{z}| \le \left|\tilde{V}_0^{-1}\right| \tilde{V}_\Delta\, |z|\,.$$

Thus, the accuracy of identification is proportional to the norm of the matrix $\tilde{V}_\Delta$ from (31).

5. Example of identification

We illustrate obtained results for solving identification problem on the following example: let matrices $A(t)$, $B(t)$, $C(t)$ be defined as

$$\begin{pmatrix} 1 & 1-\delta \\ -\delta & 1 \end{pmatrix} \le A(t) \le \begin{pmatrix} 1 & 1+\delta \\ \delta & 1 \end{pmatrix},$$

$$\begin{pmatrix} 1 & -2\delta \\ -2\delta & 1 \end{pmatrix} \le B(t) \le \begin{pmatrix} 1 & 2\delta \\ 2\delta & 1 \end{pmatrix},$$

$$\begin{pmatrix} 1-\delta & 1 \\ 0 & 1-2\delta \end{pmatrix} \le C(t) \le \begin{pmatrix} 1+\delta & 1 \\ 0 & 1+2\delta \end{pmatrix},$$

where δ be a parameter of uncertainty.

Let us fix some admitted matrices $A = \begin{pmatrix} 1 & 1+\frac{1}{2}\delta \\ \frac{1}{2}\delta & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & \delta \\ \delta & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1+\frac{1}{2}\delta & 1 \\ 0 & 1+\frac{1}{2}\delta \end{pmatrix}$. Then we take initial state $x(t_0) = x^0 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $t_0 = 0$, $t_1 = 1$, as well as real value of vector $w = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$ and construct an observation function $y(t)$.

On the second step, using obtained observation $y(t)$, we determine estimation w^* solving the linear programming problem (25). The results of calculation are represented in the Table 1.

Table 1.

δ	1	0.1	0.01	0.001
w^*	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 3.031 \end{pmatrix}$	$\begin{pmatrix} 10.024 \\ 10.238 \end{pmatrix}$	$\begin{pmatrix} 10.003 \\ 10.028 \end{pmatrix}$
ε^*	$\begin{pmatrix} 56.570 \\ 97.392 \end{pmatrix}$	$\begin{pmatrix} 4.734 \\ 3.834 \end{pmatrix}$	$\begin{pmatrix} 0.669 \\ 1.411 \end{pmatrix}$	$\begin{pmatrix} 0.066 \\ 0.138 \end{pmatrix}$

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